COURSE DESCRIPTION FOR THE CHAIR FSMP

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Algebraic local models and Galois representations

Deformation spaces of local Galois representations with p-adic Hodge theoretic conditions (the *potentially semistable deformation spaces*) are among the deepest part of the arithmetic side of the Langlands program. Progress on understanding these spaces has (via the Taylor–Wiles patching method) traditionally been the heart of some of the most striking achievements of the Langlands program such as the proof of the Taniyama-Shimura-Weil conjecture and the Sato-Tate conjecture. More recently, these deformation spaces also feature prominently in the development of the p-adic local Langlands correspondence, being the natural setting on which the correspondence should manifest.

Despite their obvious importance, our understanding of potentially semistable deformation spaces remain rudimentary. Even basic geometric properties such as integrality and regularity (essential for global applications) remain inaccessible. One of the central conjecture about these spaces is the Breuil–Mézard conjecture, which quantifies the complexity of their mod p geometry in terms of integral representation theory of p-adic groups, and which was geometrized further by Emerton and Gee by gluing different deformation spaces of varying mod p Galois representations into a *moduli stack*.

In a series of work which I obtained in various collaborations with D. Le, B. Levin and S. Morra, we develop a theory of local models for a large part of the moduli stack of Emerton–Gee. These models are of group theoretic nature, and hence appear susceptible to methods of geometric representation theory (a familiar pattern from the theory of local models of Shimura varieties). One then hopes to exploit the highly developed tools from geometric representation theory (e.g. geometry and combinatorics of affine Grassmannians, affine Springer fibers and Deligne-Lusztig theory) to understand Galois deformation spaces.

The course is aimed at giving a general introduction to this circle of ideas and accessible to an audience of Master 2 students, with emphasis on examples and explicit computations in dimension 2 and 3.