Development of Topology Optimization Method and Tool for Designing Electromagnetic Structures

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Abstract

The present work proposes Topology Optimization (TO) method to solve design problems of electromagnetic structures (EMS). These design problems involve to solve implicite inverse problems; where the purpose is to find both optimal source distribution and best material layout in topological variable region. Thus, they are formulate under the form of Non Linear Mixed-Integer Optimization Models subject to PDE-Constraint (Maxwell equation). For numerical solution, material density distribution approach called Mixed-Solid Isotropic Material with Penalization (M-SIMP) is proposed to handle efficiently such a design problem of EMS. This work is an extension of a previous one which allows to developed a home source code named $\mathtt{ATOP}^{\mathtt{TO}}$ (Algorithm To Optimize Propulsion with Topology Optimization method) for designing magnetic circuits of electrical thrusters. Some application examples dedicated to topological configuration of electromagnetic circuits for Hall effect thruster (HET) is shown and solved with success; that validated our approaches for designing EMS.

1 Statement of EMS design problem

The design domain Ω contains 3 main types of sub-domain namely: (i) a region denoted Ω_{m} for material layout, (ii) a region denoted Ω_{s} for source distribution, (iii) a target region denoted Ω_{T}. The purpose is to find the best topology $\Omega_{\mathrm{m}}^{\mathrm{opt}}$ a subset of Ω_{m} related to design parameter $\boldsymbol{p}^{\mathrm{m}}$ and the minimal value of the current densities which power the device and located in Ω_{s} related to design parameter $\boldsymbol{p}^{\mathrm{s}}$ and satisfying a imposed electromagnetic profile (or required performance) in Ω_{T}. The rest of the design domain Ω is empty (air) or occupied by component whose properties of material and source value is known denoted by Ω_{f}. Thus, the design variable is $\boldsymbol{p} = \{\boldsymbol{p}^{\mathrm{s}}, \boldsymbol{p}^{\mathrm{m}}\}$, see Figure 1.

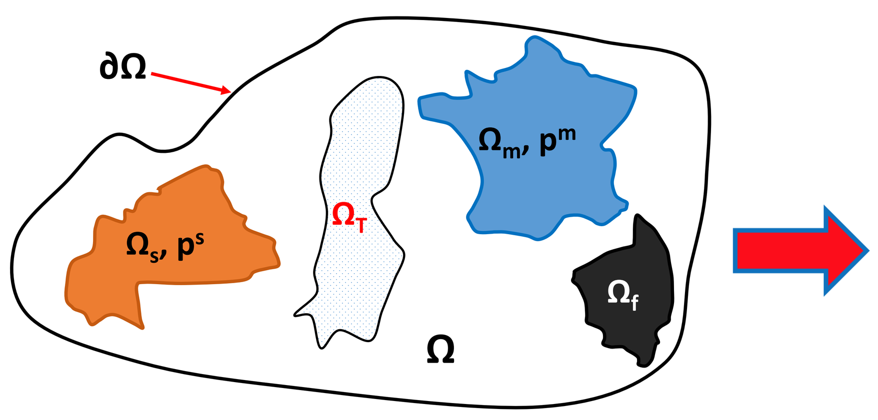
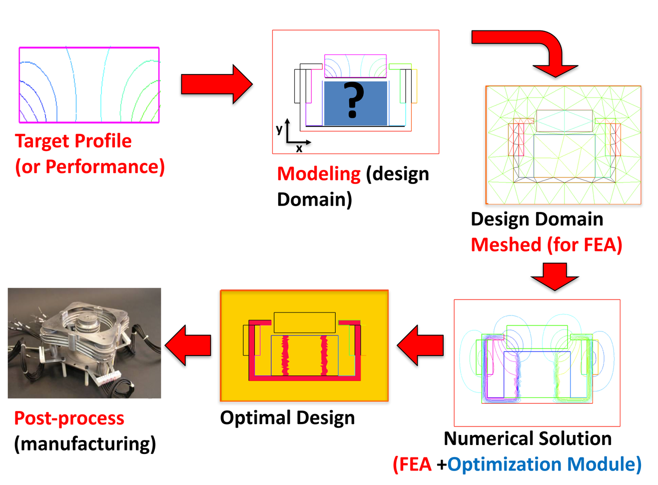


Figure 1: Design Domain subdivision and design parameters $\boldsymbol{p} = \{\boldsymbol{p}^{\mathrm{s}}, \boldsymbol{p}^{\mathrm{m}}\}$.

$$
(\mathfrak{P}) \begin{cases}
\min_{\boldsymbol{p}} F(\boldsymbol{p}) = \int_{\Omega} f(A, \boldsymbol{p}) \mathrm{d}\Omega, \\
\quad \text{s.c. :} \\
\nabla \times [\nu(\boldsymbol{p})\mathbf{B}(A)] - \boldsymbol{J}(\boldsymbol{p}) = 0, \text{ in } H_0^1(\Omega), \\
\int_{\Omega_{\mathrm{m}}} \boldsymbol{p}^{\mathrm{m}} \mathrm{d}\Omega = V_0, \\
\mathcal{I}(\boldsymbol{p}^{\mathrm{s}}) \leq 0, \\
\mathcal{E}(\boldsymbol{p}^{\mathrm{s}}) = 0, \\
\mathbf{B}(A) \in \mathcal{C}_{\Omega_{\mathrm{T}}}, \\
\boldsymbol{p} \in [\boldsymbol{p}_{\min}, \boldsymbol{p}_{\max}].
\end{cases}
$$

2 Methodological approaches

- **Optimal Design Process**



- **Main Solution Methods**
  1. **Topological Gradient** [Eschenauer, Kobelev, & Schumacher (1994), Masmoudi (1998), André & Sokolowski (2013), etc]
  2. **Level Set Method** [Osher & Sethian (1988), Jouve, Toader & Allaire (2002), etc]
  3. **Homogenization Method** [Bendsøe & Kikuchi (1988), Hassani & Hinton (1999), Allaire (2002), etc]
  4. **Density approach (SIMP)** [Bendsøe & Sigmund (1999, 2003), Rozvany (2001), etc]

3 Proposed approach: M-SIMP model

Our technique, the M-SIMP approach is a variant of the *Homogenization Method Theory* and the *classical SIMP model*. It consists of introducing a density function $\boldsymbol{\rho} := \{\boldsymbol{\rho}^{\mathrm{s}}, \boldsymbol{\rho}^{\mathrm{m}}\}$ and interpolating the design parameter $\boldsymbol{p} = \{\boldsymbol{p}^{\mathrm{s}}, \boldsymbol{p}^{\mathrm{m}}\}$ as follows: $\boldsymbol{p} = \{\boldsymbol{g}_1(\boldsymbol{\rho}^{\mathrm{s}}), \boldsymbol{g}_n(\boldsymbol{\rho}^{\mathrm{m}})\}$. Thereafter, $\boldsymbol{\rho}$ is the new design variable of the optimization problem such that for

- **source distribution parameter**,

$$\boldsymbol{p}^{\mathrm{s}}(x,y) = \boldsymbol{g}_1(\boldsymbol{\rho}^{\mathrm{s}}(x,y)) \in [\boldsymbol{p}^{\mathrm{s}}_{\min}, \boldsymbol{p}^{\mathrm{s}}_{\max}], \text{ with } \boldsymbol{\rho}^{\mathrm{s}}(x,y) \in [0,1], \forall (x,y) \in \Omega_{\mathrm{s}}. \tag{1}$$

Source distribution density function $\boldsymbol{\rho}^{\mathrm{s}}$ is **continuous on** $[0,1]$. We have:

$$\boldsymbol{g}_1(\boldsymbol{\rho}^{\mathrm{s}}) = \boldsymbol{p}^{\mathrm{s}}_{\min} + (\boldsymbol{p}^{\mathrm{s}}_{\max} - \boldsymbol{p}^{\mathrm{s}}_{\min})\boldsymbol{\rho}^{\mathrm{s}}, \quad \forall \boldsymbol{\rho}^{\mathrm{s}} \in [0,1]. \tag{2}$$

- **material layout parameter**,

$$\boldsymbol{p}^{\mathrm{m}}(x,y) = \boldsymbol{g}_n(\boldsymbol{\rho}^{\mathrm{m}}(x,y)) \in \{\boldsymbol{p}^{\mathrm{m}}_{\min}, \boldsymbol{p}^{\mathrm{m}}_{\max}\}, \forall (x,y) \in \Omega_{\mathrm{m}}, \tag{3}$$

with:

$$\boldsymbol{\rho}^{\mathrm{m}}(x,y) \in \{0,1\}, \ \forall (x,y) \in \Omega_{\mathrm{m}}, \quad \text{s.t.} \quad \boldsymbol{\rho}^{\mathrm{m}}(x,y) = \begin{cases} 1 \text{ if } (x,y) \in \Omega_{\boldsymbol{p}^{\mathrm{m}}_{\max}}, \\ 0 \text{ if } (x,y) \in \Omega_{\boldsymbol{p}^{\mathrm{m}}_{\min}}. \end{cases} \tag{4}$$

Material layout determination density function $\boldsymbol{\rho}^{\mathrm{m}}$ is **binary design variable**. We have:

$$\boldsymbol{g}_n(\boldsymbol{\rho}^{\mathrm{m}}) = \boldsymbol{p}^{\mathrm{m}}_{\min} + \frac{\boldsymbol{p}^{\mathrm{m}}_{\max} - \boldsymbol{p}^{\mathrm{m}}_{\min}}{n} \sum_{i=1}^{n} (\boldsymbol{\rho}^{\mathrm{m}})^i, \quad \forall \boldsymbol{\rho}^{\mathrm{m}} \in [0,1] \quad \text{(after relaxation)}. \tag{5}$$

⇒ M-SIMP formulation of $(\mathfrak{P})$ is a Non Linear Mixed-Integer PDE-Constrained Optimization Problem depending only on the design variable $\boldsymbol{\rho} := \{\boldsymbol{\rho}^{\mathrm{s}}, \boldsymbol{\rho}^{\mathrm{m}}\} \in [0,1] \times \{0,1\}$.

Numerical optimization method is described as following:

- **Step 1: Relaxation of** ρ^{m} on $[0,1]$ ⇒ Implementation of **Continuous Optimization Algorithms**.
  - **Advantage**: Use of **Gradient Based Algorithms** which are most efficient on large size problems. In our work, the *topological derivative is computed* by *Adjoint Variable Method* (in Structurale Optimization, that takes the form of ***Design Sensitivity Analysis***).
  - **Difficulty**: Appearance of **Intermediate Values** (IVs) namely when $\rho^{\mathrm{m}} \notin \{0,1\}$ in the optimal solution. In fact these non binary optimal values for material distribution variable induce porous, composite or non existant material in the computed topologies.
- **Step 2: Penalization of IVs**: Introduction of *heuristic methods* to penalize emergence of IVs in the optimal design ⇒ During optimization process, ρ^{m} is constrained to be more closed to 0 or 1 as possible (hence the acronym **SIMP**: **S**olid **I**sotropic **M**aterial with **P**enalization of IVs).

Question: How to penalize these IVs ?

– A judicious choice of the parameter n known as the ***penalization degree/factor*** related to the function $\boldsymbol{g}_n$ with a smart way to update it in the optimization loop, see Figure 2 and reference [1].

4 Developed Tool: $\mathtt{ATOP}^{\mathtt{TO}}$

$\mathtt{ATOP}^{\mathtt{TO}}$ is a set of programs for designing electromagnetic circuits with TO method based on material distribution approaches. It is built with well-known scientific computation softwares: FEMM 4.2 (Finite Element Method Magnetics version 4.2) developed by David Meeker (2010) for Finite Element Analysis (FEA) and MATLAB (by MathWorks society) for optimization module of the program. Architecture of $\mathtt{ATOP}^{\mathtt{TO}}$ can be summarized on the diagram in Figure 2 (More details in [1]).

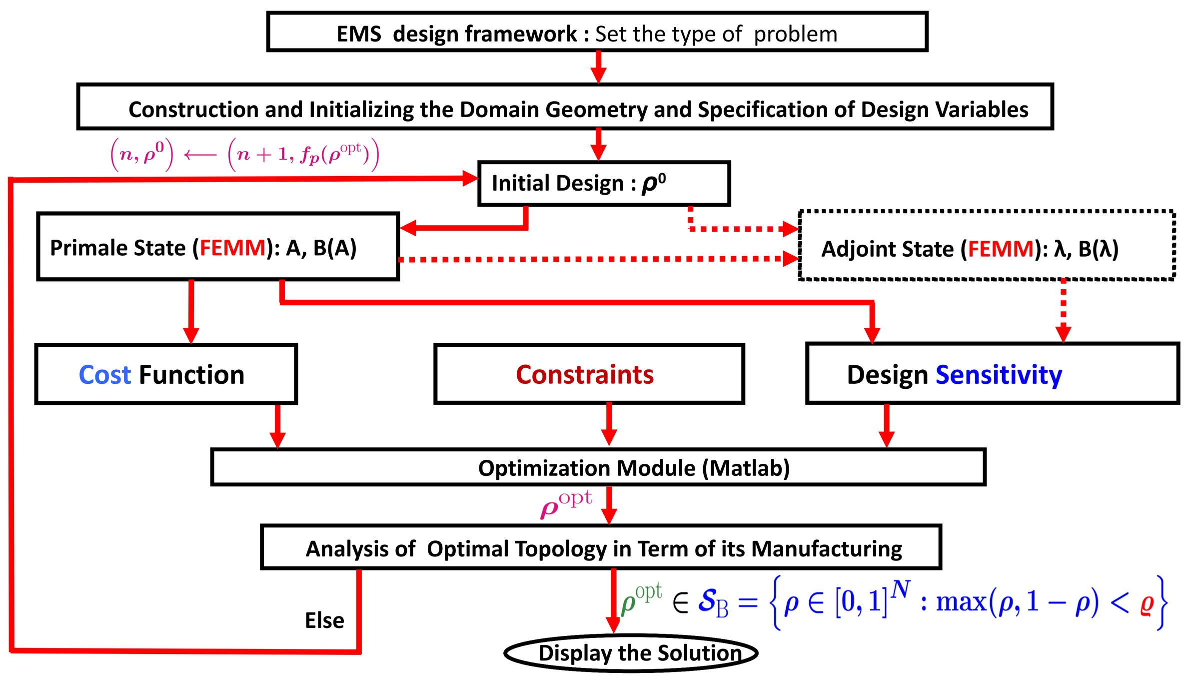


Figure 2: Principle of functioning of $\mathtt{ATOP}^{\mathtt{TO}}$. The part of the flowchart in dotted line is the finite element analysis step of the adjoint state (when that is necessary). Primal and Adjoint State are PDE-Constraint of design problem, see $(\mathfrak{P})$.

5 Applications: Design of Hall Effect Thruster (HET)

$f(A,\boldsymbol{p}) = A\boldsymbol{J}(\boldsymbol{p})$, $\boldsymbol{p}^{\mathrm{s}}$: current density $\boldsymbol{J}$, $\boldsymbol{p}^{\mathrm{m}}$: magnetic permeability $\boldsymbol{\mu}$, penalty factor : $n = 50$.

- **Cross section of a HET**

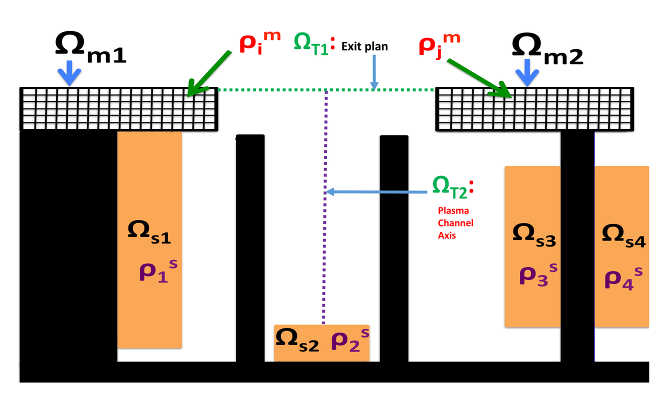


Figure 3: Discretized domain: the design variable $\boldsymbol{\rho} = (\rho_1^{\mathrm{s}}, \dots, \rho_4^{\mathrm{s}}, \rho_1^{\mathrm{m}}, \dots, \rho_{900}^{\mathrm{m}}) \in \mathbb{R}^{904}$; $\Omega_{\mathrm{m}} = \Omega_{\mathrm{m1}} \cup \Omega_{\mathrm{m2}}$, $\Omega_{\mathrm{s}} = \Omega_{\mathrm{s1}} \cup \Omega_{\mathrm{s2}} \cup \Omega_{\mathrm{s3}} \cup \Omega_{\mathrm{s4}}$, $\Omega_{\mathrm{T}} = \Omega_{\mathrm{T1}} \cup \Omega_{\mathrm{T2}}$ and Ω_{f} is black part.

- **Problem setting**
  - Design parameter bounds for :
  - ◦ Sources: $\{\boldsymbol{p}^{\mathrm{s}}_{\min}, \boldsymbol{p}^{\mathrm{s}}_{\max}\} = \{-3, +3\}$ (A/mm^2),
  - ◦ Material: $\{\boldsymbol{p}^{\mathrm{m}}_{\min}, \boldsymbol{p}^{\mathrm{m}}_{\max}\} = 4\pi 10^{-7}\{1, 8000\}$ (H/m).
  - Number of design variables: $N = 4(\textbf{Sources}) + 2 \times 30 \times 15(\textbf{Ferromagnetic})$.
  - Linear constraints for:
  - ◦ Sources: $\rho_3^{\mathrm{s}} = \rho_4^{\mathrm{s}}$, and $\sum \rho_i^{\mathrm{s}} \leq 2$,
  - ◦ Material: $\sum \rho_i^{m} = 40\%\mathrm{vol}(\Omega_{\mathrm{m}})$.
  - Constraints of magnetic field $\mathbf{B}$:

$$\mathcal{C}_{\Omega_{\mathrm{T}}} = \left\{ \mathbf{B} : \left| \tan\left(\frac{\mathrm{B}y}{\mathrm{B}x}\right) \right| \leq 0.18 \text{ in } \Omega_{\mathrm{T1}};\ \|\mathbf{B}\| \leq 0.04 \text{ in } \Omega_{\mathrm{T2}} \right\}.$$

Numerical results

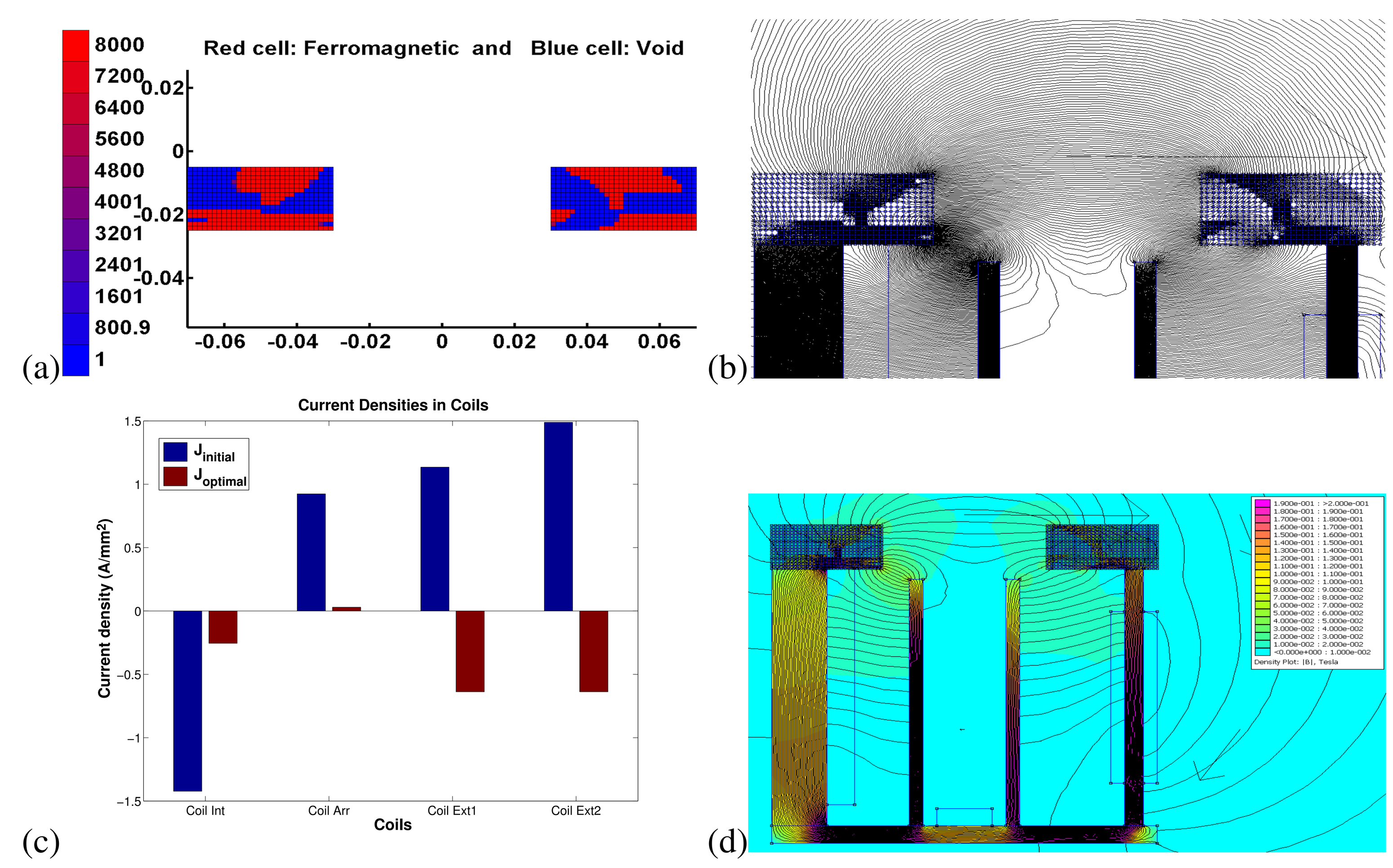


Figure 4: For this numerical application test: The cost function varies from the value $F^0 = 2.38$ to $F^{\mathrm{opt}} = 0.99$, in 138 iterations; (a) Optimal topology, (b) Magnetic induction flow through the optimal circuit shape, (c) Optimal source distribution in coils, (d) Magnetic field intensity distribution in the computed design domain.

Conclusions and Perspectives

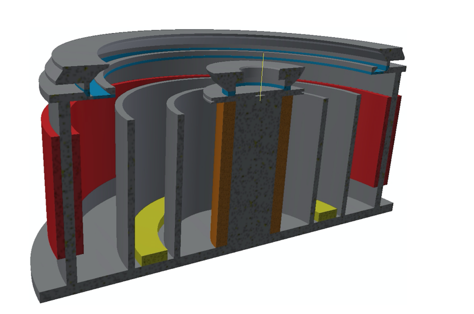


Figure 5: Additive manufacturing: illustration of the corresponding HET electromagnetic circuit of the obtained optimal solution.

TO based on M-SIMP approach is suitable for EMS design problems in engineering applications where several control parameters must be considered. The program $\mathtt{ATOP}^{\mathtt{TO}}$ is an efficient and robuste optimal design tool. These works help for automatic, rational and optimal design procedure and make it possible to take best decision for manufacturing EMS devices by reducing time and cost of production.

Hybridation Technique: Apply Shape Optimization on the optimal topologies (smoothing technique).

References

[1] S. Sanogo, *Conception Optimale de Circuits Magnétiques dédiés à la Propulsion Spatiale Électrique par des Méthodes d'Optimisation Topologique*, PhD-thesis, LAPLACE, UPS-Toulouse III, France, Février 2016.